DEF

Linearly Dependent & Independent (Algebraic)

The vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ are ***linearly dependent*** if there is a non-trivial linear combination of $\vec{v}_1, \ldots, \vec{v}_n$ that equals the zero vector. Otherwise they are linearly independent.

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21.1 Explain how the geometric definition of linear dependence (original) implies this algebraic one (new).

21.2 Explain how this algebraic definition of linear dependence (new) implies the geometric one (original).

Since we have geometric def $\implies$ algebraic def, and algebraic def $\implies$ geometric def ($\implies$ should be read aloud as 'implies'), the two definitions are *equivalent* (which we write as algebraic def $\iff$ geometric def).

22 Suppose for some unknown $\vec{u}, \vec{v}, \vec{w}$, and $\vec{a}$,

$$\vec{a} = 3\vec{u} + 2\vec{v} + \vec{w} \qquad \text{and} \qquad \vec{a} = 2\vec{u} + \vec{v} - \vec{w}.$$

22.1 Could the set $\{\vec{u}, \vec{v}, \vec{w}\}$ be linearly independent?

Suppose that

$$\vec{a} = \vec{u} + 6\vec{r} - \vec{s}$$

is the *only* way to write $\vec{a}$ using $\vec{u}, \vec{r}, \vec{s}$.

22.2 Is $\{\vec{u}, \vec{r}, \vec{s}\}$ linearly independent?

22.3 Is $\{\vec{u}, \vec{r}\}$ linearly independent?

22.4 Is $\{\vec{u}, \vec{v}, \vec{w}, \vec{r}\}$ linearly independent?

Linear Independence and Dependence, Creating Examples

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1. Fill in the following chart keeping track of the strategies you used to generate examples.

	Linearly independent	Linearly dependent
A set of 2 vectors in $\mathbb{R}^2$		
A set of 3 vectors in $\mathbb{R}^2$		
A set of 2 vectors in $\mathbb{R}^3$		
A set of 3 vectors in $\mathbb{R}^3$		
A set of 4 vectors in $\mathbb{R}^3$		

2. Write at least two generalizations that can be made from these examples and the strategies you used to create them.